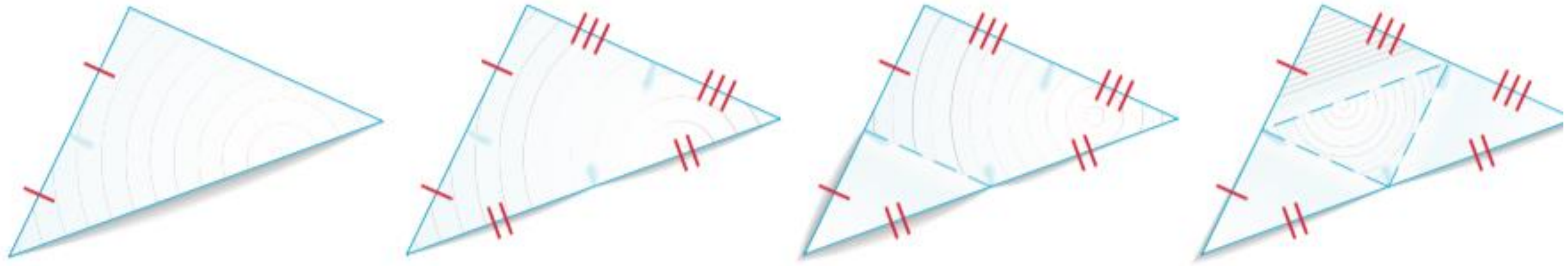
Chapter 5
Properties of Triangles

Section 4
Midsegment Theorem

GOAL 1: Using Midsegments of a Triangle

In Lessons 5.2 and 5.3, you studied four special types of segments of a triangle: perpendicular bisectors, angle bisectors, medians, and altitudes. Another special type of segment is called a midsegment. A midsegment of a triangle is a segment that connects the midpoints of two sides of a triangle.

You can form the three midsegments of a triangle by tracing the triangle on paper, cutting it out, and folding it, as shown below.



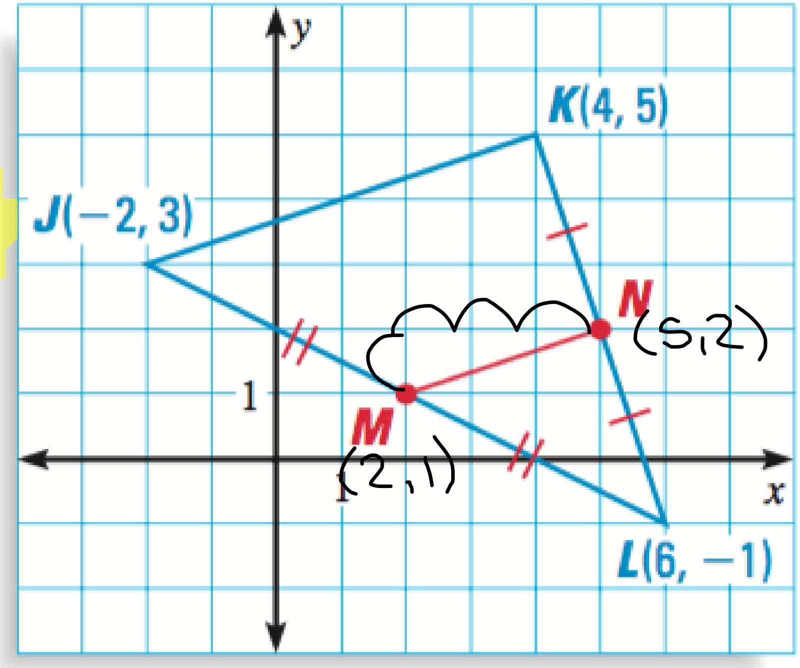
1. Fold one vertex onto another to find one midpoint.
2. Repeat the process to find the other two midpoints.
3. Fold a segment that contains two of the midpoints.
4. Fold the remaining two midsegments of the triangle.

Example 1: Using Midsegments

$\sqrt{(x-x)^2+(y-y)^2}$

same slope | distance formula

Show that the midsegment $\overline{MN}$ is parallel to side $\overline{JK}$ and is half as long.



Parallel

Slope MN: $\frac{1}{3}$; Slope JK: $\frac{2}{6} \rightarrow \frac{1}{3}$

are parallel

1/2 as long

MN: $\sqrt{(5-2)^2+(2-1)^2} \rightarrow \sqrt{9+1} \rightarrow \sqrt{10} \rightarrow 3.16$

JK: $\sqrt{(4-{}^-2)^2+(5-3)^2} \rightarrow \sqrt{36+4} \rightarrow \sqrt{40} \rightarrow 6.32$

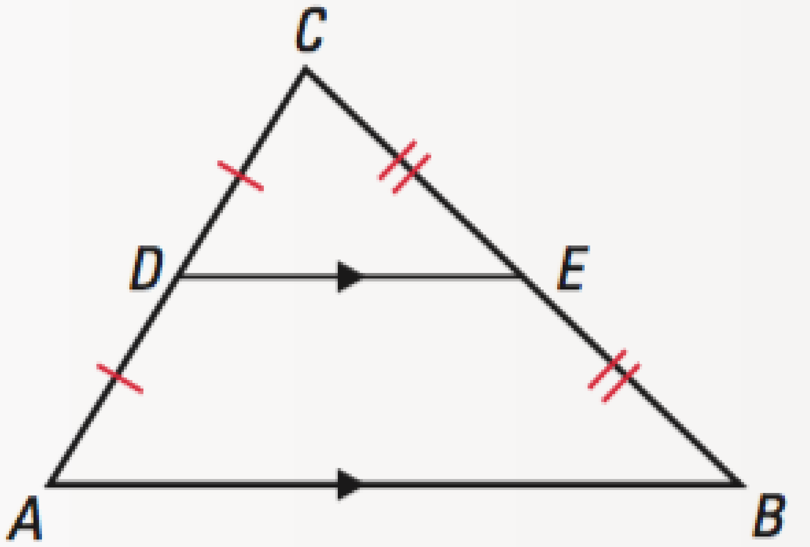
MN is 1/2 of JK

THEOREM

THEOREM 5.9 ***Midsegment Theorem***

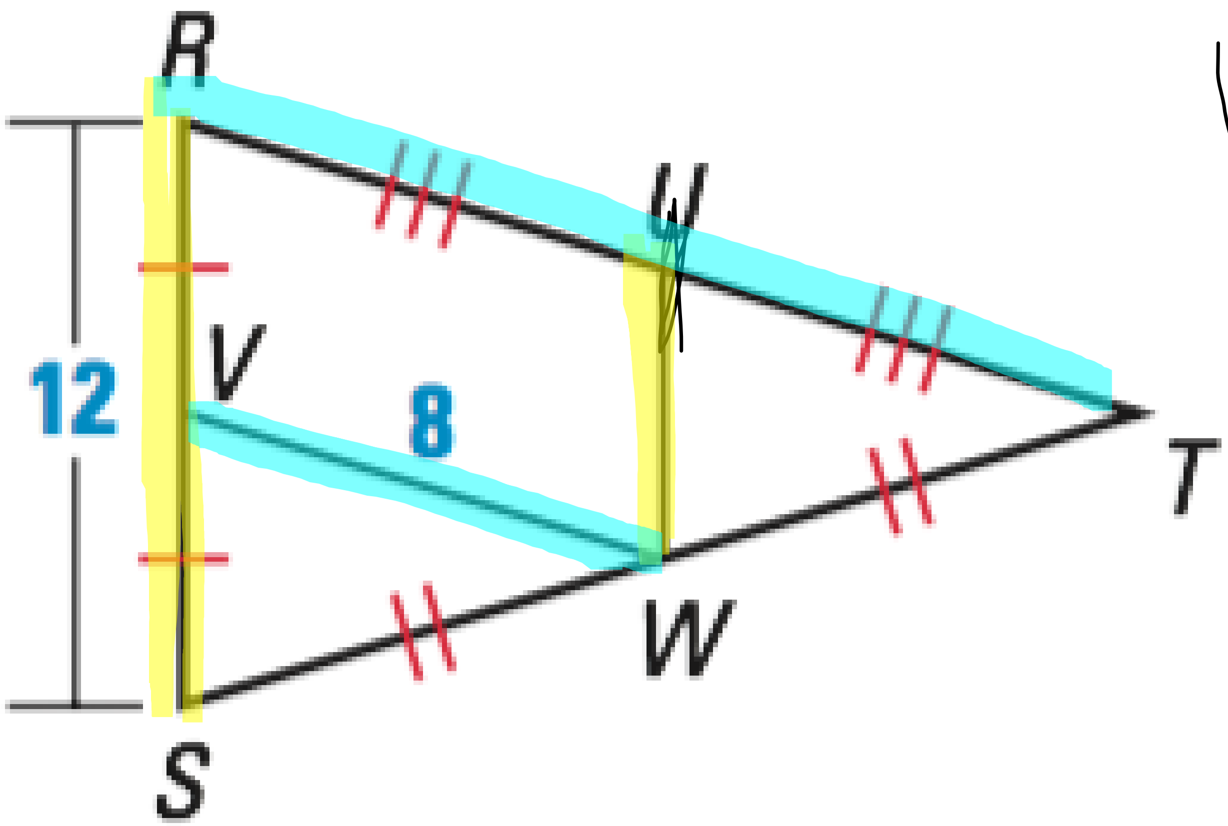
The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long.

$$\overline{DE} \parallel \overline{AB} \text{ and } DE = \frac{1}{2}AB$$



Example 2: Using the Midsegment Theorem

$\overline{UW}$ and $\overline{VW}$ are midsegments of ΔRST. Find UW and RT.



$UW = 1/2(12)$
$= 6$

$RT = 2(8)$
$= 16$

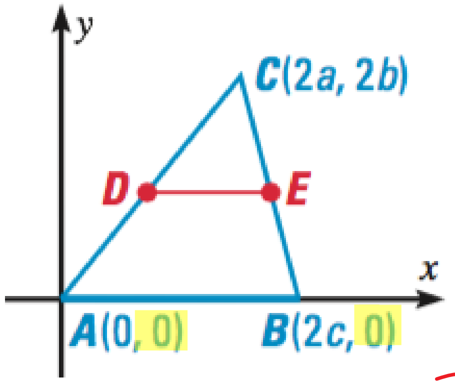
Example 3: Proving Theorem 5.9

$$\left(\frac{x+x}{2}, \frac{y+y}{2}\right)$$

$$\frac{2a+2c}{2} \rightarrow \frac{\cancel{2}(a+c)}{\cancel{2}}$$

Write a coordinate proof of the Midsegment Theorem.

(Hint: Find coordinates of D and E. Find slope of DE and AB. Find lengths of AB and DE.)



D: $\left(\frac{2a+0}{2}, \frac{2b+0}{2}\right) \rightarrow \left(\frac{2a}{2}, \frac{2b}{2}\right) \rightarrow (a, b)$

E: $\left(\frac{2a+2c}{2}, \frac{2b+0}{2}\right) \rightarrow (a+c, b)$

DE → slope = 0

AB → slope = 0

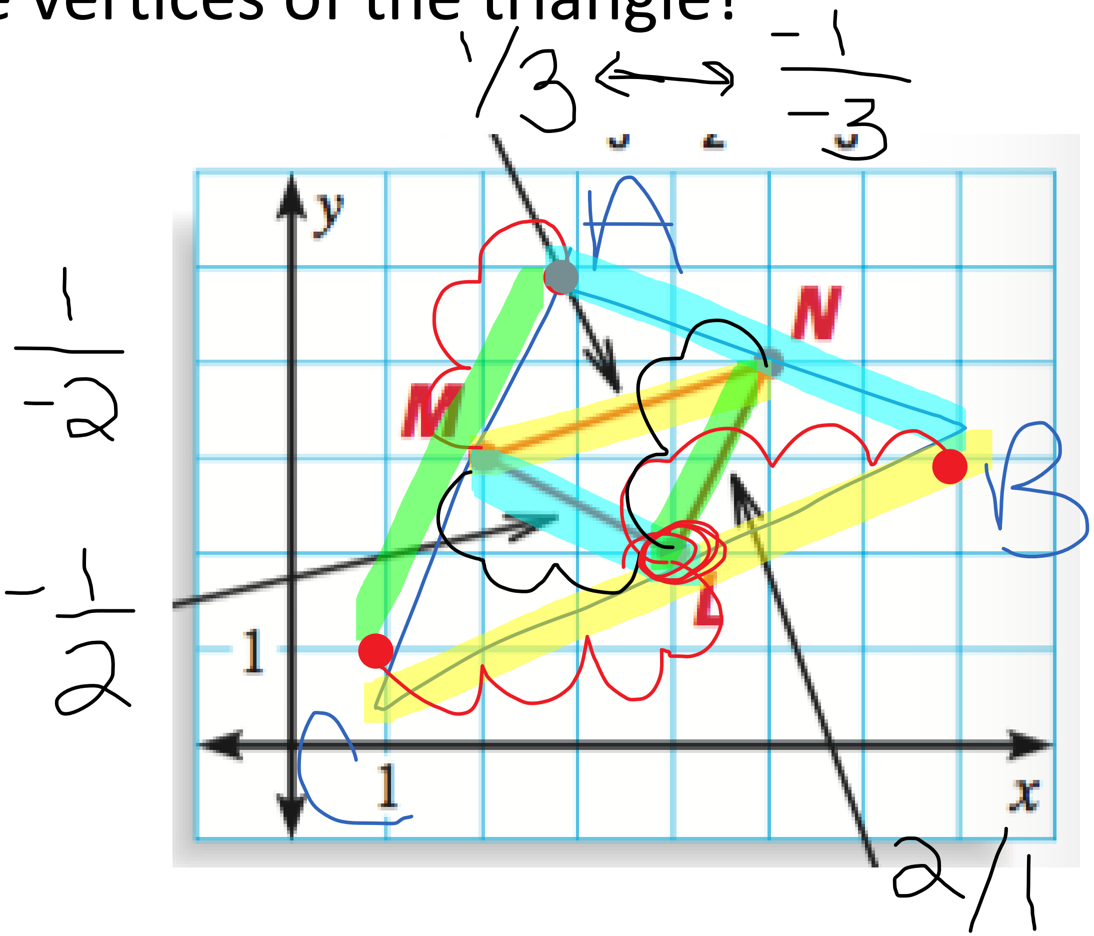
DE → $\sqrt{(a-a+c)^2+(b-b)^2} \rightarrow \cancel{\sqrt{c^2}} \rightarrow c$

AB → $\sqrt{(2c-0)^2+(0-0)^2} \rightarrow \cancel{\sqrt{(2c)^2}} \rightarrow 2c$

GOAL 2: Using Properties of Midsegments

Example 4: Using Midpoints to Draw a Triangle

The midpoints of the sides of a triangle are L(4, 2), M(2, 3), and N(5, 4). What are the coordinates of the vertices of the triangle?

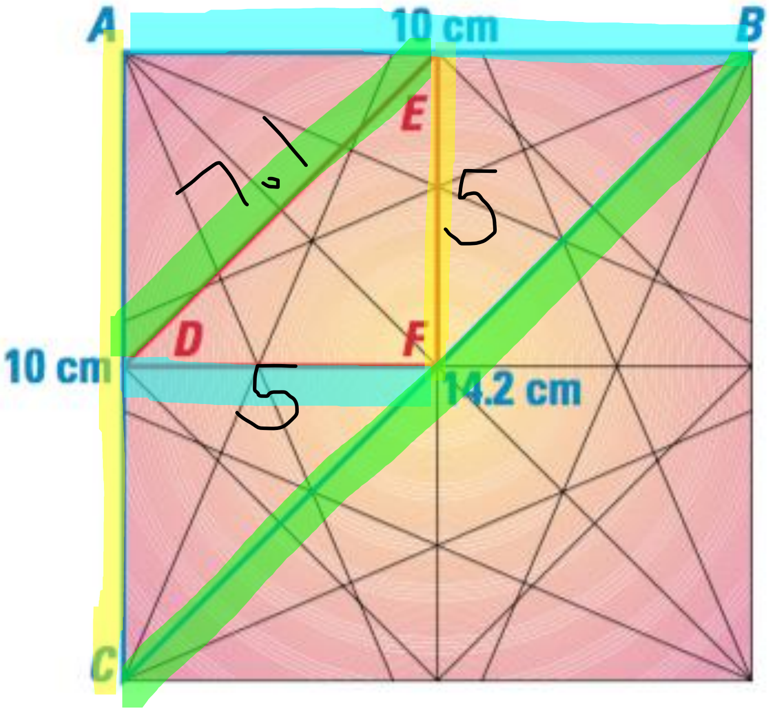


A(3,5)
B(7,3)
C(1,1)

The perimeter of the triangle formed by the three midsegments of a triangle is half the perimeter of the original triangle.

Example 5: Perimeter of Midsegment Triangle

$\overline{DE}$, $\overline{EF}$, and $\overline{DF}$ are midsegments in ΔABC. Find the perimeter of ΔDEF.



$5+5+7.1$

$=17.1 \text{ cm}$

$10+10+14.2$

$=34.2/2=17.1$

EXIT SLIP